The Time-Frequency Interpretation for Transient Evolution of Wave Propagation through Dispersive Medium

Dong Xiaoting, Jiang Yansheng, and Wang Wenbing

***Abstract*—The propagation of electromagnetic waves in a linear, dispersive Lorentz medium is calculated using finite-difference time-domain (FDTD) method; their time-frequency (TF) characteristics are studied using Gabor extension. Numerical results show that TF spectrum gives clear interpretation for transient evolution of ultra wide-band pulse propagation through Lorentz medium.**

***Index Terms*—Dispersive medium, time-frequency analysis, transient electromagnetic field.**

I. INTRODUCTION

THE SPECTRUM of propagated ultra-wideband pulse through dispersive Lorentz medium is time variation due to the frequency-dependent group velocity. Precursors and distortion are introduced, which make the time domain signal difficult to identify. Recently, time-frequency (TF) analysis has been used to handle and has shown strong potential for interpreting nonstationary signal. Earlier researches show us the possibility to obtain the group-delay curve of the dispersive medium in the time-frequency spectrum; results of waves in perfect waveguide or cold plasma have been obtained [1]. But for dispersive medium with absorption, such as Lorentz medium, the group velocity in abnormal dispersive range is greater than the vacuum speed of light c, although no portion of the field could propagate so fast [2]; TF characteristics of wave propagation have not been mentioned. Study of this problem can help to analyze the echo of ultra-wideband radar, and to separate, then extract the object and medium characteristics.

In this paper, propagated time domain signals are simulated first using finite-difference time-domain (FDTD) method; results of FFT method are also presented to check up the validity of FDTD accuracy. Then, with these results, TF spectrums are studied using Gabor extension. By comparison with the frequency-group delay curve of Lorentz medium, the TF characteristics are interpreted successfully, and feasibility of confirming the arrival-time and extracting medium parameters are further discussed.

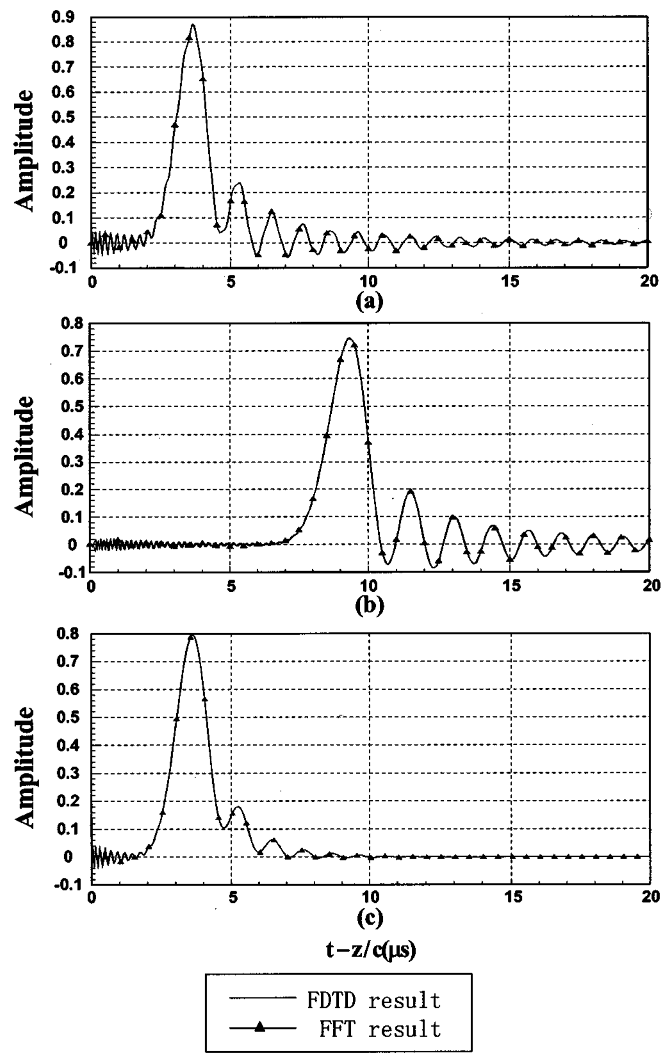

Fig. 1. Propagated double exponential pulse through Lorentz medium with parameters of (a) $\omega_c = 1 \times 10^7$/s, $b = 1 \times 10^7$/s, $\delta = 1 \times 10^5$/s; (b) $\omega_c = 1 \times 10^7$/s, $b = 2 \times 10^7$/s, $\delta = 1 \times 10^5$/s; (c) $\omega_c = 1 \times 10^7$/s, $b = 1 \times 10^7$/s, $\delta = 5 \times 10^5$/s. The propagation distance is 2000 m. Comparison of FDTD results with FFT results.

II. FREQUENCY-APPROXIMATION MODELS FOR FDTD SIMULATION

In the time domain, the polarization response can be approximated in the form of a convolution integral [3]

$$D(t) = \int_{-\infty}^{t} \varepsilon(t-\tau) E(\tau)\, d\tau. \tag{1}$$

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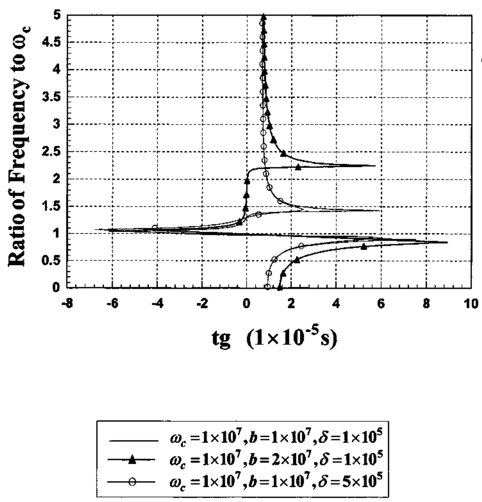

Fig. 2. Frequency-group delay curve of different Lorentz medium at propagation distance of 2000 m. The coordinate of y-axis is the ratio of frequency to the medium parameter ω_c.

To calculate fields at a certain time using FDTD need not only fields of the prestep, but also of all the steps before. This leads to a linear increscent storage requirement. To solve this problem, obtaining the difference equation from the frequency model of material dispersion is needed. Various models of handling specific types have been developed. A technique of permitting the implementation for an nth-order dispersive model that adds only n real storage variables per point to the standard FDTD algorithm was given by Hulse [4]. His approach is used in our FDTD implementation; the update equation of the electronic field can be expressed as a polynomial of the following:

$$E^{k+1}(l) = \left[\sum_{i=0}^{n} b_i D^k(l-i) - \sum_{i=1}^{n} a_i E^k(l-i)\right] \Big/ a_0 \quad (2)$$

where a_i, b_i $i = 0, n$ are the update coefficients. Fig. 1 shows the propagation of double exponential pulses in Lorentz medium with different parameters. The original signal is denoted as

$$g(t) = A_0[\exp(-\alpha_1 t) - \exp(-\alpha_2 t)]u(t), \qquad 0 < \alpha_1 < \alpha_2 \quad (3)$$

where

$A_0 = 1.435$;
$\alpha_1 = 1 \times 10^6$;
$\alpha_2 = 1 \times 10^7$;

and the step of FDTD grids is $dt = 1 \times 10^{-8}$s, $dz = c \times dt$. The propagation distance is $z = 2000$ m. Simulated results using FFT method [2] are also presented in Fig. 1 to check up the validity of FDTD accuracy. Results of the two methods inosculate well.

III. Frequency-Group Delay Curve of Dispersive Medium

For a linear dispersive Lorentz medium with one resonance frequency ω_c, the complex index of refraction is given by

$$n(\omega) = \left(1 - \frac{b^2}{\omega^2 + 2j\delta\omega - \omega_c^2}\right)^{1/2} \quad (4)$$

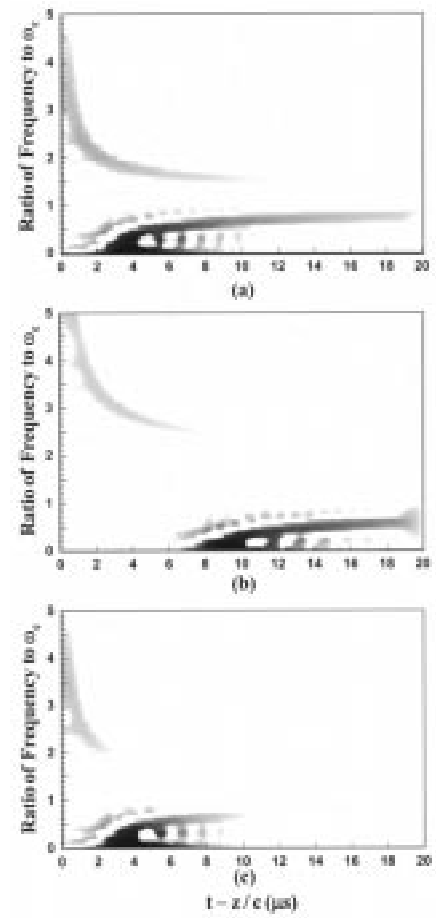

Fig. 3. TF spectrum of propagated double exponential pulse through Lorentz medium, with parameters of (a) $\omega_c = 1 \times 10^7$/s, $b = 1 \times 10^7$/s, $\delta = 1 \times 10^5$/s; (b) $\omega_c = 1 \times 10^7$/s, $b = 2 \times 10^7$/s, $\delta = 1 \times 10^5$/s; (c) $\omega_c = 1 \times 10^7$/s, $b = 1 \times 10^7$/s, $\delta = 5 \times 10^5$/s. The propagation distance is 2000 m.

and the frequency-dependent group-delay of Lorentz medium can be represented as

$$t_g(\omega) = \mathrm{Re}\left\{n(\omega) - \frac{\omega b^2(-2\omega + 2j\delta)}{2n(\omega)(\omega_c^2 - \omega^2 + 2j\delta\omega)^2}\right\}\frac{z}{c}. \quad (5)$$

The group-delay curves of Lorentz medium with different parameters are shown in Fig. 2; the frequency is limited from 0 to $5 \times \omega_c$, and t_g is limited from -100 to 100 μs.

IV. Results and Interpretations

The TF spectrums of three propagated double exponential pulses demonstrated in Fig. 1 are shown in Fig. 3. As we can see, the first precursor arrives at $t = z/c$; its frequency is very high at the arrival time and then decreases to $\sqrt{\omega_c^2 + b^2}$. The second precursor arrives later at $t = (z/c)(\sqrt{\omega_c^2 + b^2})$, as time continues the frequency increasing from zero. These are coincident with the results from asymmetric method [5]. In fact, classical analysis of the time-domain signal has utilized the concept of instantaneous frequency, and the signal was divided into several parts and considered separately.

We also remark that the declining parameter δ is relevant to the existent time of frequency components, the larger the δ, the shorter duration. The tendency of TF characteristics is coherent with the frequency group-delay curve as medium parameters

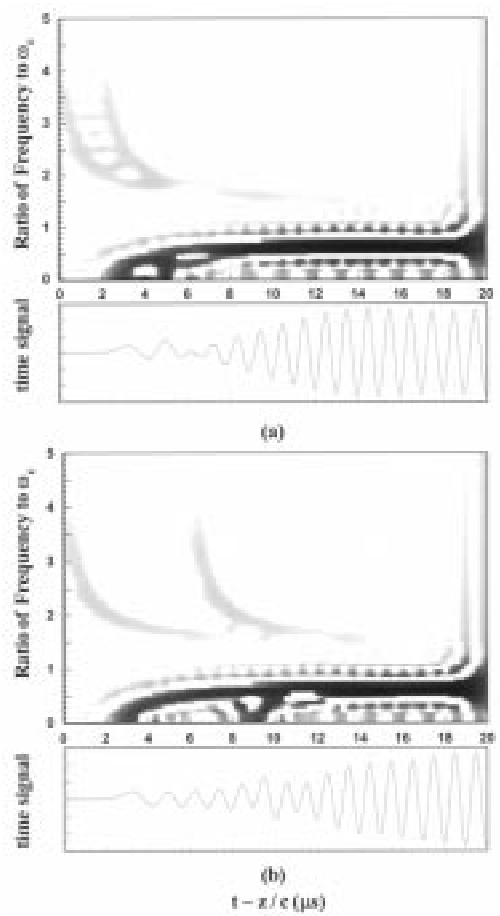

Fig. 4. TF spectrum and time-domain signal of two unit step-function modulated signal with a time interval of (a) 2 μs; (b) 6 μs. The carrier frequency is $2 \times \pi \times 10^6$/s. Medium parameters are $\omega_c = 1 \times 10^7$/s, $b = 1 \times 10^7$/s, and $\delta = 1 \times 10^5$/s. The propagation distance is 2000 m.

changing (shown in Fig. 2). These imply the possibility to obtain the medium parameters from the TF spectrum or the instantaneous parameters of the propagated signal. Furthermore, the group velocity in the abnormal regime is greater than c; in fact, no portion of the fields could propagate so fast. Signal energy is zero in the TF spectrum, relatively.

It is known that in the dispersive medium, correct description of the signal arrival is closely associated with the proper analysis of the precursor fields and their role in the dynamical evolution of the field, but the amplitude of the precursor fields are insignificant in comparison to the amplitude of the signal, and it is usually hardly to find out the arrival time from the time-domain signal. Fortunately, from TF spectrum, the arrival time is easy to find due to the high frequency of the first precursor; we can distinguish two signals even if the second signal arrives at the dynamic evolution of the first signal (the distinguishable time interval is limited by the wideness of time window we used). As shown in Fig. 4, the propagation and TF spectrums of the addition of two unit-step-function modulated sine signal arriving with a time interval of (a) 2 μs and (b) 6 μs. In Fig. 4(a), the second signal arrives at the dynamic evolution of the first signal, and in Fig. 4(b), the second signal arrives at later time while the first signal has reached its stationary status. In both of them, arrival times of the two signals are well distinguished. The TF spectrum provides a clear description of the entire dynamical field evolution.

V. Conclusion

Numerical results show that the TF spectrum gives clear interpretation for transient evolution of ultra-wideband pulse propagation through dispersive medium, such as the first and second precursors. The classical characteristics of the TF spectrum are coincident with those in the frequency group-delay curve of dispersive medium. Also, the TF spectrum helps to distinguish the arrival time of signal.

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